A new CP violation mechanism generated by the standard neutral Higgs boson : the $\eta \to \pi + \pi$ case.

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Abstract

Strictly within the standard electro-weak interaction, CP violation in the flavour conserving process $\eta \to \pi + \pi$ could originate from the mixing of the η meson with the virtual scalar Higgs boson H^0 via W and top quark exchange.

The parity-violation carried by weak gauge bosons makes the mixing possible by quantum effect at two-loop level. Nowhere the Kobayashi-Maskawa (KM) phase mechanism is needed. The phenomenon reveals an unexpected new role of the Higgs boson in the CP symmetry breaking.

For the Higgs mass between 100-600 GeV, the $\eta \to \pi + \pi$ branching ratio is found to be $3.6 \cdot 10^{-26} - 2.4 \cdot 10^{-29}$, hence CP violation mechanisms beyond the Standard Model are the only ones that could give rise to its observation at existing or near future η factories, unless the Higgs mass is improbably as light as 550 MeV.

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To understand the origin and the nature of CP violation, in addition to the studies of flavour changing K and B mesons processes, investigations are also needed in flavour conserving ones [1] for which the $\eta \to \pi + \pi$ decay and the electric dipole moment of baryons are some typical examples. Like the K_L^0, an eventual coexistence of both three and two-pion decay modes of the η would imply that CP is violated in the flavour conserving sector. Therefore experimental searches for the two-pion decay mode of the η is of great interest [1], and the purpose of this letter is two fold :

1. point out an unexpected new role of the Higgs boson in the CP breaking, generated by quantum effect.

2. give a reliable estimate of its branching ratio.

We do find indeed that CP violation in $\eta \to \pi + \pi$ can be triggered by the neutral Higgs boson, *independently of all other mechanisms* [2, 3, 4, 5] : the decay can occur *not by explicit CP non conservation* term put by hand in the lagrangian (like the standard KM non-zero phase assumed from the start), but by a completely different mechanism through the mixing between the η and the neutral Higgs boson.

This point is basically new, it can only occur at two loop level as shown in Figs.1,2,3 using the renormalizable gauge R_ξ : besides the $W^\pm, Z^0$ gauge bosons, the would-be $\chi^\pm, \chi^3$ Goldstone-Higgs fields (those absorbed by $W^\pm, Z^0$ to get masses) also contribute. They represent the complete set of two loop diagrams that participate to the η-Higgs mixing. At one loop level, the mixing cannot take place, there is no way to get rid of the γ_5 coupling of the η with its antisymmetric tensor $i\varepsilon_{\mu\nu\rho\sigma}$, while the Higgs coupling is symmetric.

The main reason for the mixing to occur is the following: the γ_5 coupling of the η with quarks when combined with the product $V \times A$ of the gauge boson couplings will be absorbed and give rise to a symmetric tensor $g_{\mu\nu}$ and makes the mixing with the Higgs boson possible.

Since both the η meson and the Higgs boson have $C = +1$, physically it means that through two loop integrals, the parity-violation $V \times A$ property of the weak bosons can shift the intrinsic $P = -1$ of the η into the $P = +1$ of the H^0, hence CP $-1 \leftrightarrow +1$ mixing : *parity-violation turns out to be the source of CP non-conservation, due to the Higgs and gauge bosons interplay*. This observation is first illustrated by explicit computation of the Figure 1 diagram, the relevant quantity to be considered is:

$$I(k^2) \equiv (-1) \int \frac{d^4q}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \frac{Trace[\gamma_5(\not{p}+m)\gamma_\alpha(1-\gamma_5)(\not{q}+m')\gamma_\beta(1-\gamma_5)(\not{p}+\not{k}+m)]g^{\alpha\beta}}{(q^2-m'^2)(p^2-m^2)((p+k)^2-m^2)((p-q)^2-M^2)((p+k-q)^2-M^2)} \tag{1}$$

with $m = m_s, m' = m_c, M = M_W$. The contributions of other graphs will be given later. In Eq.(1) we have taken, as an illustrative example, the $s\bar{s}$ component of the η in the loop.

An important remark is in order : Unlike the $W^+ + W^-$ exchange in Fig. 1, for the $Z^0 + Z^0$ one, there are in fact two identical diagrams with fermion circulates in opposite directions around the loop, their contributions give rise to only antisymmetric tensor and make the mixing with the Higgs boson impossible. The reason is that ZZ are identical particles (similar to $\eta \to \gamma\gamma$ case). However for the W^+W^- case, there is only one diagram. To check this point one can go back to the T products $T(H_\eta(x)H_W(y)H_W(z))$, apply the Wick theorem and rediscover the Feynman rules and graphs. Here $H_\eta = \eta\bar{s}\gamma_5 s$, $H_W = \bar{c}\gamma_\mu(1 - \gamma_5)s \; W^\mu + h.c.$. We can easily understand this point even with tree diagrams, let us compare $s + \bar{s} \to Z^0 + Z^0$ to $s + \bar{s} \to W^+ + W^-$.

To compute Eq.(1) we first integrate over d^4p using the x, y, z Feynman parametrization, and then over d^4q. Let us sketch out the sucessive steps. After calculating the trace, we obtain:

$$I(k^2) = 16m \int \frac{d^4q}{(2\pi)^4} \frac{kq}{(q^2 - m'^2)} J(k, q) \tag{2}$$

where :

$$
\begin{aligned}
J(k, q) &= \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)((p+k)^2 - m^2)((p-q)^2 - M^2)((p+k-q)^2 - M^2)} \\
&= \frac{i\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^x dy \int_0^y \frac{dz}{(x-z)^2(1-x+z)^2} \frac{1}{(q^2 + 2qK - L^2)^2}
\end{aligned} \tag{3}
$$

$$K = k\frac{z(1-y) - y(1-x)}{(x-z)(1-x+z)} \quad , \quad L^2 = \frac{M^2(x-z) - m^2(1-x+z) + k^2y(1-y)}{(x-z)(1-x-z)}, \tag{4}$$

When we neglect m^2, m'^2, k^2 compared to M^2, then we find:

$$\int \frac{d^4q}{(2\pi)^4} \frac{kq}{(q^2 - m'^2)(q^2 + 2qK - L^2)^2} = i\pi^2 \frac{k^2}{2M^2} \frac{z(1-y) - y(1-x)}{x-z} + \mathcal{O}\left(\frac{k^4}{M^4}, \frac{m^2}{M^2}, \frac{m'^2}{M^2}\right) \tag{5}$$

such that

$$I(k^2) = C \; \frac{m}{32\pi^4} \; \frac{k^2}{M^2}[1 + \mathcal{O}\left(\frac{k^2}{M^2}, \frac{m^2}{M^2}, \frac{m'^2}{M^2}\right)] \tag{6}$$

where

$$C \equiv \int_0^1 dx \int_0^x dy \int_0^y dz \frac{y(1-x) - z(1-y)}{(x-z)^3(1-x+z)^2} = \frac{1}{4} \tag{7}$$

The computation of C in Eq.(7) is tedious, numerical integration is helpless since there is a delicate cancellation of infinities. We have done it analytically by hand, step by step. The result is C=1/4. The expression (6) we obtain for the two-loop integration is impressively simple, because higher orders in $\frac{k^2}{M^2}$, $\frac{m^2}{M^2}$, $\frac{m'^2}{M^2}$ (beyound the linear term $\frac{k^2}{M^2}$) are neglected in the course of our d^4q integration.

In the d^4p one, everything is kept however. Without this legitimate approximation, we would obtain an avalanche of - unnecessary and numerically negligible- complicated expressions involving, among others, the dilogarithmic (or Spence) $\mathcal{L}_2$ function frequently met in such circumstance.

The diagram 1 looks like the familiar triangle in $\pi^0 \to 2\gamma$ with the external gauge bosons momenta integrated. Immediately a question arises whether or not our *convergent and finite $I(k^2)$ term* has something to do with the possible anomalous Ward identity [6] via $k^\mu P_\mu(k^2) \equiv Y(k^2)$ where $P_\mu(k^2)$ is defined similarly to $I(k^2)$ of Eq.(1) in which the first γ_5 at the extreme left of the numerator of Eq.(1) is replaced by $\gamma_\mu \gamma_5$. The interrelation between $I(k^2)$ and $P_\mu(k^2)$ is respectively similar to that between the pseudotensors $R_{\alpha\beta}(k^2)$ and $T_{\mu\alpha\beta}(k^2)$ intervened in $\pi^0 \to 2\gamma$. We find out, after the trace calculation, that $Y(k^2)$ could be written as

$$Y(k^2) = 16 \int \frac{d^4q}{(2\pi)^4} \frac{F(q,k)}{q^2 - m'^2} \tag{8}$$

where after the integration over d^4p, $F(q,k)$ is found to be

$$F(q,k) = \frac{i\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^x dy \int_0^y \frac{dz}{(x-z)^2(1-x+z)^2} \frac{N(q,k)}{(q^2 + 2qK - L^2)^2} \tag{9}$$

with

$$N(q,k) = qk\{-2q^2(x-z)^2 + 2qk[z(1-y)-y(1-x)] - (M^2-m^2)(x-z) + 2k^2y(1-y)\} - q^2k^2(x-z)(1-2y) \tag{10}$$

As expected, the integral over d^4q in Eq. (8) is formally divergent, and in the regularization procedure one might think that if one tries to imitate the $\pi^0 \to 2\gamma$ case, then by a careless shift of the integration p, q variables, one would rediscover an analogy with the false Sutherland-Veltman theorem [7] while a careful shift would lead to an analogy with the ABJ anomaly [6]. However the analogy stops here, since contrary to the $\pi^0 \to 2\gamma$ case for which there is always one γ_5 with the tensor $\varepsilon_{\mu\nu\alpha\beta}$ source of the ABJ anomaly, here in our case *the γ_5 does not intervene*, the quantities $P_\mu(k^2)$ and $I(k^2)$ are respectively vectors and scalar objects, contrary to the corresponding pseudotensors $T_{\mu\alpha\beta}(k^2)$ and $R_{\alpha\beta}(k^2)$ in the $\pi^0 \to 2\gamma$ case, from that chiral anomaly was discovered. It is interesting to note that when we cut the diagram 1 at the two W^+, W^- lines, i.e. we compute the one loop triangle $\eta \to W_\alpha^+ + W_\beta^-$, we get the imaginary part Im $I(k^2) = 0$, thus checking by a dispersion relation that $I(k^2)$ is real, as we have already obtained before in Eq.(6).

We now go on to the other diagrams. For the diagram 2, the calculation is more complicated than the diagram 1, and we get result similar to Eq.(6) with C replaced by

$$D_t \equiv \rho(1 + \frac{\rho}{2})|V_{ts}|^2 D(\rho) \tag{11}$$

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where

$$D(\rho) = \frac{1}{(\rho-1)^2} + \frac{\rho}{4(\rho-1)^3} + \frac{\rho(2\rho-1)}{2(\rho-1)^3}\log\rho$$
$$+ \frac{\rho^2}{(\rho-1)^2}\left[\mathcal{L}_2(\frac{\rho-1}{\rho}) - \frac{\pi^2}{6}\right] + \frac{\rho}{(\rho-1)^3}\int_0^1 dx(1-2x)\mathcal{L}_2(\frac{x+\rho-1}{\rho}). \tag{12}$$

and $\rho = m_t^2/M_W^2$.

The latter integration can be also analytically computed in terms of products of the logarithmic function, its expression is rather combersome and not given here. Also we have computed all other contributions denoted collectively by E. Numerically, it turns out that the diagram 1 is dominant.[3]

It remains two questions to be settled: the first one concerns the effective point-like coupling constant $g_{\eta Q\overline{Q}}$ of the η meson with quarks Q assumed in Figs. 1,2,3. Is it justified ? The second point deals with the off-shell (virtually light mass $k^2 = m_\eta^2$) Higgs decay amplitude into two pions.

1- The justification for the η-quarks coupling can be traced back to its Goldstone nature, to the partially conserved axial current (PCAC) and its consequence : the Goldberger-Treiman (GT) relation. Its well known application is the $\eta, \pi \to 2\gamma$ rate that gives the number of colors to be three.

2- The virtually light ($k^2 = m_\eta^2$) Higgs boson coupling to two pions can be reliably estimated from the so-called conformal anomaly i.e. the trace of the energy- momentum tensor in QCD[8] $\Theta_\mu^\mu = -\beta_0\frac{\alpha_s}{8\pi}G_{\mu\nu}G^{\mu\nu}$ ($\beta_0 = 9$ is the first coefficient of the QCD β function). The crucial point-as explained in [9] - is that the matrix element of the operator $\alpha_s G_{\mu\nu}G^{\mu\nu}$ between the two-pion state and the vacuum is nonvanishing in the chiral limit, it even does not depend on α_s ; the (virtually light k^2) Higgs decay amplitude into two pions is found to be [9] :

$$f_{H\pi\pi}(k^2) = -\frac{g}{\beta_0}\ \frac{k^2 + 5.5\ m_\pi^2}{M_W} \tag{13}$$

where $g = e/sin\theta_W$ is the standard SU(2) gauge coupling which enters also in the three other vertices of Figs. 1,2,3. Putting altogether the ingredients, we obtain for the $\eta \to \pi + \pi$ decay amplitude the following result :

$$A_{\eta\pi^+\pi^-} = A_{\eta\pi^0\pi^0} = \frac{1}{6\sqrt{3}}\ \left(\frac{G_F M_W^2}{4\pi^2}\right)^2\ \frac{m_\eta^2}{M_W^2}\ \frac{m_\eta^2 + 5.5m_\pi^2}{m_H^2 - m_\eta^2}\ \frac{X}{f_\eta}\ \left(1 + \frac{D_t + E}{C}\right) \tag{14}$$

with

$$X = m_s^2(\sqrt{2}cos\theta_P + sin\theta_P) - (m_u^2 + m_d^2)\left(\frac{cos\theta_P}{\sqrt{2}} - sin\theta_P\right) \tag{15}$$

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[3]The details of calculations will be given elsewhere, contributions of all other two loop diagrams are negligible: the up, top quark contributions in Fig. 1, the up, charm quark as well as the Z^0, χ^3 in Fig. 2. As for the diagram 3, it is even smaller.

from which :

$$\Gamma(\eta \to \pi^+\pi^-) = 2 \ \Gamma(\eta \to \pi^0\pi^0) = \frac{|A_{\eta\pi\pi}|^2}{16\pi m_\eta} \ \sqrt{1 - \frac{4m_\pi^2}{m_\eta^2}} \tag{16}$$

In Eqs. (14) -(15), the GT like relation $g_{\eta Q\overline{Q}} = m_Q/f_\eta$ is used, quark color indices are summed up, and $\theta_P \simeq -19^o$ is the flavour SU(3) $\eta - \eta'$ mixing angle determined from their two photon rates. We take $f_\eta = f_\pi \simeq 93 \ MeV$. It turns out that the numerical values of the quantity $Y \equiv X/f_\eta$ entering in Eq. (14) are relatively insensitive to the choices of quark masses : for the constituent ones $m_s = 500 \ MeV$, $m_u = m_d = 300 \ MeV$, we have $Y = 0.79 \ GeV$; for the current ones $m_s = 200 \ MeV$, $m_u = m_d = 8 \ MeV$, we get $Y = 0.44 \ GeV$. With the constituent mass choice, we obtain :

$$Br(\eta \to \pi + \pi) = 3.6 \ \cdot \ 10^{-26} \ (\frac{100 \ GeV}{M_H})^4 \tag{17}$$

such that for the Higgs mass between $100 \ GeV$ and $600 \ GeV$, the branching ratio into both charged and neutral pions of the η meson varies in the range $3.6 \cdot 10^{-26} - 2.4 \cdot 10^{-29}$, which is similar although somewhat larger than the Jarlskog and Shabalin (JS) [5] result (for the Higgs mass $\leq 250 GeV$).

Therefore the standard model predicts that existing as well as future η factories (Saturne, Celsius, Daphne) could not detect the $\eta \to \pi + \pi$ mode (unless the Higgs mass is improbably as light as $550 \ MeV$), implying that *unconventional* CP violation mechanisms are the only ones that could give rise to its eventual observation. This fact is not as negative as it seems, since as noted by JS, new CP violation mechanisms, what ever they may be, will have a golden opportunity to show up in the $\eta \to \pi + \pi$ decay. *Its eventual observation in η factories would definitely rule out the standard CP violation mechanism (like KM or our mixing), its experimental search is even more interesting for this reason.*

In conclusion, we point out that CP violation in η decay can occur, *not by explicit CP non conservation term put by hand in the lagrangian, but by its mixing with the Higgs boson through quantum effect.* It is interesting on its own right. Conceptually the phenomenon may share a common point (but not to get confused) with the ABJ anomaly in chiral lagrangian : the chiral symmetry with massless u, d quarks has a conserved axial current, however due to quantum loop effect, the latter is no more conserved. Also is the induced θ term with strong CP odd in QCD. In our case, the KM phase is not needed, the interaction is CP conserving, however, induced by quantum loop, a mixing between *states with opposite parities* can occur and trigger a CP violating effect, in which an unexpected new role of the Higgs boson is revealed.

Acknowledgements

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Figure Captions :

1. Figure 1 : η - Higgs mixing by two-loop WW, $\chi\chi$, and $W\chi$ exchange. U_i denotes u, c, t quarks, only the charm quark contribution is important.

2. Figure 2 : η - Higgs mixing by two-loop U_i quark exchange via W and χ. U_i denotes u, c, t quarks, only the top quark contribution is important.

   Similar diagrams with Z^0, χ^3 replacing W and $\chi^\pm$ turn out to be negligible.

3. Figure 3 : η - Higgs mixing by two-loop via W and χ exchange. U_i denotes u, c, t quarks.

   Similar diagrams with Z^0, χ^3 replacing W and $\chi^\pm$ with $U_i = s$.